Kaon B-parameter using Overlap Fermions

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I present first results from an in-progress calculation of B_K in quenched approximation using overlap fermions. My particular implementation of the overlap uses a kernel with nearest and next-nearest neighbor interactions and HYP-blocked gauge connections. Matching to the continuum NDR regularization is done perturbatively. I present preliminary results at $\beta = 5.9$ and 6.1 (lattice spacings 0.125 and 0.09 fm) for quark masses, pseudoscalar decay constants, and B-parameter – $B_K^{(NDR)}(\mu = 2\ \text{GeV}) \simeq 0.66(3-4)$.

The kaon B-parameter B_K, defined as $\frac{8}{3}(m_K f_K)^2 B_K = \langle \bar{K}|\bar{s}\gamma_\mu(1-\gamma_5)d\bar{s}\gamma_\mu(1-\gamma_5)d|K\rangle$, has been computed many times with lattice methods. Lattice calculations of B_K require actions with good chiral properties, to prevent operator mixing with wrong-chirality operators from contaminating the signal. There has been a continuous cycle of lattice calculations using fermions with ever better chiral properties. This calculation is yet another incremental upgrade, to the use of a lattice action with exact $SU(N_f) \otimes SU(N_f)$ chiral symmetry, an overlap action. These actions have operator mixing identical to that of continuum-regulated QCD.

The overlap action used in these studies[1] is built from a kernel action with nearest and next-nearest neighbor couplings, and HYP-blocked links[2]. HYP links fatten the gauge links without extending gauge-field-fermion couplings beyond a single hypercube. This improves the kernel's chiral properties without compromising locality. The kernel action is designed to resemble the exact overlap well enough that its eigenvectors are good "seeds" for a calculation of eigenvectors of the exact action, and it is kept simple enough that finding its own eigenvectors is inexpensive. These eigenvectors are used to precondition the calculation of quark propagators, in principle eliminating *all* critical slowing down at small quark mass.

The data set is generated in the quenched approximation using the Wilson gauge action at couplings $\beta = 5.9$ (on a $12^3 \times 36$ site lattice) and $\beta = 6.1$ (on a $16^3 \times 48$ site lattice) with 40 lattices each (so far). The nominal lattice spacings are $a = 0.125$ fm and 0.090 fm from the measured rho mass. Propagators for six quark masses are constructed corresponding to pseudoscalar-to-vector meson mass ratios of $m_{PS}/m_V \simeq 0.6$ to 0.85.

Table 1
Results from these simulations.

	$\beta = 5.9$	$\beta = 6.1$
$1/a$ (MeV)	1580(60)	2190(140)
$m_{nonstrange}$ (MeV)	4.3(3)	4.5(3)
$m_{strange}$ (MeV)	105(5)	110(7)
m_s/m_{ns}	24.40(4)	24.41(5)
f_π (MeV)	142(11)	131(12)
f_K (MeV)	155(10)	147(11)
$B_K^{(NDR)}(\mu = 2$ GeV)	0.66(3)	0.66(4)
$B_K^{(RGI)}$	0.92(4)	0.93(6)

The methodology for B_K is well-developed: compute an un-amputated correlator which contains the desired matrix element (two kaon sources far apart on the lattice with the four-fermion operator sandwiched in between), clip off the $(m_K f_K)^2$ prefactor by simultaneously measuring the matrix element $\langle 0|\bar{s}\gamma_0\gamma_5 d|K\rangle$, extrapolate/interpolate the lattice $B-$parameter to its value at the kaon mass, and convert the lattice number to its continuum-regulated counterpart.

To maximize the signal volume I computed propagators from two well-separated sources ($N_t/2-2$ temporal sites apart) and brought them together to the operator. I used Gaussian sources to maximize overlap onto the ground state. These sources do not make momentum eigenstates, and so the $p = 0$ B_K signal is contaminated by a $\vec{p} \neq 0$ contribution. This causes problems at bigger quark mass, because $E(p) - m$ gets smaller as the pseudoscalar mass m grows. Fortunately, there are two inequivalent paths on the torus to disentangle the two "signals," and one can fit the B_K correlator to a sum of a $\vec{p} = 0$ term and a $p = 2\pi/N_s$ term. This is not a problem at small quark mass.

I extracted a signal from fits to the traditional ratio of the $\bar{K} - K$ amplitude and product of two-point functions, as well as correlated fits to the $\bar{K} - K$ amplitude and the two-point functions. A ratio plot is shown in Fig. 1. A typical correlated fit is shown in Fig. 2, and a plot of lattice B_K vs. quark mass is shown in Fig. 3. Uncorrelated jackknife "ratio fits" have small uncertainties and are quite stable over a wide range of timeslices. However, one would really like to do correlated fits. The data points are strongly correlated, and it is necessary to use singular value decomposition to invert the correlation matrix. When I do this, I find that my fits are consistent with the jackknife fits and have reasonable confidence levels. To get to the physical kaon I linearly extrapolated my results with a jacknife; there is no sign of discernable curvature in my data.

I calculated the renormalization factors between the lattice- and continuum-regulated (NDR) matrix elements using one loop perturbation theory. At a (lattice spacing) $\times\mu$ (continuum regularization point) $= 1$, a conversion factor is $Z = 1 + (\alpha_s(q^*)/4\pi)z$. As one might expect from related work[3], the HYP link pushes the constant z close to zero. The cost is that q^* (defined a la Lepage-Mackenzie[4]) can move to a small value, but sensible values of q^* are given by the higher-order prescription of [5].

For B_K, the operator O_+ has a matching factor Z_+ and for the overlap action used here, its parameter $z = -4.0$ at $q^*a = 0.92$ for NDR. The entire conversion factor for B_K from lattice $\beta = 5.9$ to $\mu = 2$ GeV is $Z_+/Z_A^2 = 0.99$. (Wilson-action kernel overlap actions have z's which are an order of magnitude larger.)

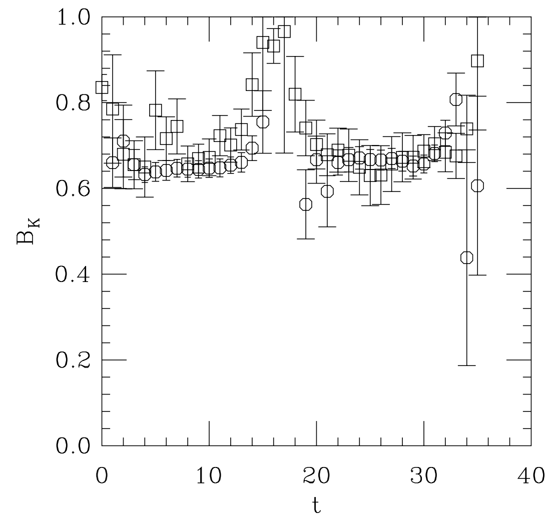

Figure 1. A traditional ratio plot of the B_K graph divided by the product of two point graphs, from the $\beta = 5.9$ data set at quark mass $am_q = 0.050$ with axial current sources and sinks (squares) and pseudoscalar sources and sinks (octagons).

How reliable is this number? I have not checked it directly (yet), but perturbation theory for the local axial vector current can be tested with overlap actions by a comparison of the vacuum-to-pseudoscalar meson matrix elements of the axial vector and pseudoscalar density. At $\beta = 5.9$ I find $Z_A = 0.97$ or 0.98 (depending on the choice of lowest-order or higher-order q^*, and 0.97(1) nonperturbatively. Perturbation theory for the matching coefficient fo the $\overline{MS}$ quark mass can also be compared to the nonperturbative calculation of Ref. [6]. This analysis gives $Z(\mu = 2\ \mathrm{GeV}, \mathrm{a}) = 1.10(3)$ at $r_0 m_{PS} = 5$ and 1.14(11) at $r_0 m_{PS} = 3$, as compared to the perturbative prediction of 0.95.

My PRELIMINARY values for B_K and for other relevant parameters (lattice spacings from rho mass, decay constants, $\overline{MS}(\mu = 2$ GeV) quark masses) are recorded in the Table. A combined error from fitting, extrapolation, and lattice spacing (dominated by statistics) is shown. My B_K result is in reasonably good agreement

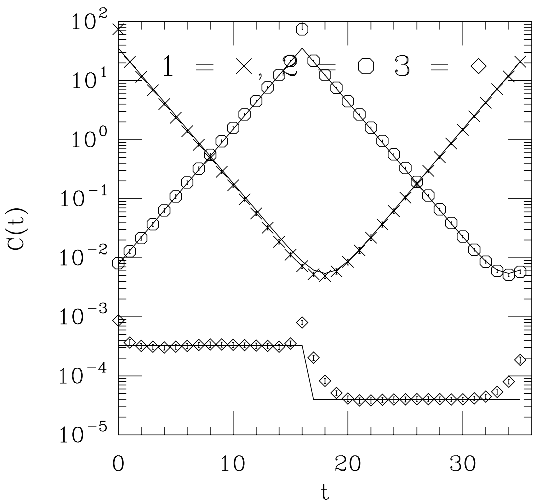

Figure 2. The two axial current correlators (labeled "1" and "2") and the "figure-eight" correlator (labeled "3"), for the $am_q = 0.100$ $\beta = 5.9$ data set with axial current sources. A correlated fit to three correlators over the range $t = 7 - 9$ and 24-28 is also shown.

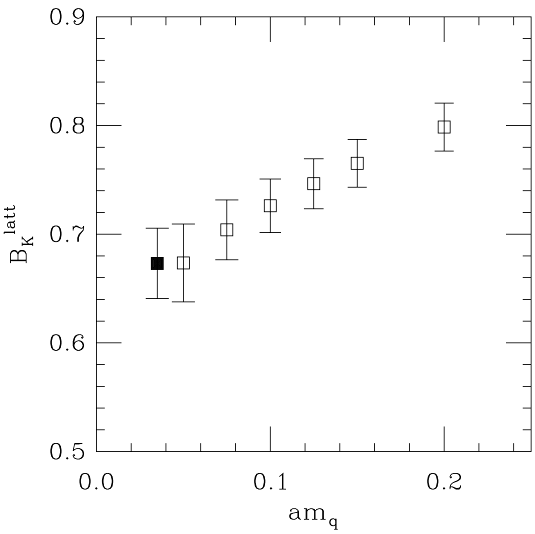

Figure 3. Lattice B_K (from ratio fits) at $\beta = 5.9$, with a jackknifed linear extrapolation to the kaon (half the strange quark mass) (solid symbol).

with the staggered JLQCD result[7] a bit higher than the CP-PACS[8] domain wall fermion result and quite a bit higher than the RBC[9] domain wall fermion result. It is also consistent with the Wilson-overlap results presented by Lellouch[10] at this meeting. Of course, it is a linear extrapolation: that is a dangerous thing to do. If the allocation gods allow, I hope to push to smaller quark masses (bracketing the necessary quark mass and possibly revealing the chiral logarithm) and collect more statistics (hopefully giving more respectable error bars).

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REFERENCES

1. T. DeGrand [MILC collaboration], Phys. Rev. D **63**, 034503 (2001) [hep-lat/0007046].
2. A. Hasenfratz and F. Knechtli, Phys. Rev. D **64**, 034504 (2001) [arXiv:hep-lat/0103029].
3. C. W. Bernard and T. DeGrand, Nucl. Phys. Proc. Suppl. **83**, 845 (2000); W. j. Lee and S. R. Sharpe, arXiv:hep-lat/0208018, arXiv:hep-lat/0208036.
4. G. P. Lepage and P. B. Mackenzie, Phys. Rev. D **48**, 2250 (1993) [arXiv:hep-lat/9209022].
5. K. Hornbostel, G. P. Lepage and C. Morningstar, Nucl. Phys. Proc. Suppl. **94**, 579 (2001) [arXiv:hep-lat/0011049].
6. P. Hernandez, K. Jansen, L. Lellouch and H. Wittig, JHEP **0107**, 018 (2001) [arXiv:hep-lat/0106011].
7. S. Aoki *et al.* [JLQCD Collaboration], Phys. Rev. Lett. **80** (1998) 5271 [arXiv:hep-lat/9710073].
8. A. Ali Khan *et al.* [CP-PACS Collaboration], Phys. Rev. D **64** (2001) 114506 [arXiv:hep-lat/0105020].
9. T. Blum *et al.* [RBC Collaboration], arXiv:hep-lat/0110075.
10. L. Lellouch, this volume.